TRINITY COLLEGE

Mathematics Specialist Units 1,2
Test 2 2018

Section 1 Calculator Free

Vectors

STUDENT'S NAME SOLUTIONS

DATE: Thursday 29 March **TIME:** 20 minutes **MARKS**: 23

INSTRUCTIONS:
Standard Items: Pens, pencils, drawing templates, eraser

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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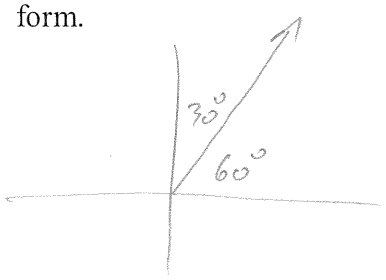
1. (6 marks)

(a) A vector has a magnitude of 8 and a direction of 030°. Express the vector in component form. [3]



$x = 8\cos 60^\circ$
$= 4$
$y = 8\sin 60^\circ$
$= 4\sqrt{3}$

$(4, 4\sqrt{3})$

(b) Point R has a position vector of $\begin{pmatrix} -3 \\ 5 \end{pmatrix}$. Vector $\overrightarrow{RT} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$. Determine the position vector of point T. [3]

$\overrightarrow{RT} = \overrightarrow{OT} - \overrightarrow{OR}$
$\begin{pmatrix} 2 \\ 3 \end{pmatrix} = \overrightarrow{OT} - \begin{pmatrix} -3 \\ 5 \end{pmatrix}$
$\begin{pmatrix} -1 \\ 8 \end{pmatrix} = \overrightarrow{OT}$

2. (8 marks)

Given $\underset{\sim}{a} = 5i - 12j$ and $\underset{\sim}{b} = i + 3j$, determine:

(a) $|\underset{\sim}{a} - \underset{\sim}{b}|$ [2]

$\begin{pmatrix} 5 \\ -12 \end{pmatrix} - \begin{pmatrix} 1 \\ 3 \end{pmatrix}$

$= \begin{pmatrix} 4 \\ -15 \end{pmatrix}$

$|a - b| = \sqrt{4^2 + 15^2}$

$= \sqrt{241}$

(b) $\hat{\underset{\sim}{b}}$ [2]

$\frac{i + 3j}{\sqrt{1^2 + 3^2}}$

$= \frac{i + 3j}{\sqrt{10}}$

(c) a vector in the direction of $\underset{\sim}{a} + \underset{\sim}{b}$ with the magnitude of $2\underset{\sim}{b}$ [4]

$\begin{pmatrix} 5 \\ -12 \end{pmatrix} + \begin{pmatrix} 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 6 \\ -9 \end{pmatrix}$

$\left|\begin{pmatrix} 6 \\ -9 \end{pmatrix}\right| = \sqrt{36 + 81}$

$= \sqrt{117}$

$\left|\begin{matrix} 2 \\ 6 \end{matrix}\right| = \sqrt{2^2 + 6^2}$

$= \sqrt{40}$

VECTOR $= \frac{\sqrt{40}(6i - 9j)}{\sqrt{117}}$

3. (9 marks)

(a) Given $\underset{\sim}{a} = 3i - 2j$, $\underset{\sim}{b} = 5i + 2j$ and $\underset{\sim}{c} = 2i + tj$, determine

(i) the value of t if $\underset{\sim}{c}$ is parallel to $3\underset{\sim}{a} - \underset{\sim}{b}$ [2]

$3\begin{pmatrix}3\\-2\end{pmatrix} - \begin{pmatrix}5\\2\end{pmatrix}$

$= \begin{pmatrix}4\\-8\end{pmatrix}$

$\begin{pmatrix}2\\t\end{pmatrix} = \lambda\begin{pmatrix}4\\-8\end{pmatrix}$

$2 = 4\lambda$ $\quad t = \frac{1}{2}(-8)$

$\frac{1}{2} = \lambda$ $\quad = -4$

(ii) the value of t if $-\underset{\sim}{a} + 3\underset{\sim}{b}$ is perpendicular to $\underset{\sim}{c}$ [2]

$\begin{pmatrix}-3\\+2\end{pmatrix} + 3\begin{pmatrix}5\\2\end{pmatrix}$

$= \begin{pmatrix}12\\8\end{pmatrix}$

$\begin{pmatrix}12\\8\end{pmatrix} \cdot \begin{pmatrix}2\\t\end{pmatrix} = 0$

$24 + 8t = 0$

$t = -3$

(b) Given $\underset{\sim}{m} = xi + yj$, $|\underset{\sim}{m}| = \sqrt{113}$, $\underset{\sim}{n} = \begin{pmatrix}2\\3\end{pmatrix}$ and $\underset{\sim}{m} \bullet \underset{\sim}{n} = 10$, determine x and y if $y > 0$. [5]

$\sqrt{x^2 + y^2} = \sqrt{113}$

$x^2 + y^2 = 113$

$\begin{pmatrix}x\\y\end{pmatrix} \cdot \begin{pmatrix}2\\3\end{pmatrix} = 10$

$2x + 3y = 10$

$x = -7$

$y = 8$

TRINITY COLLEGE

Mathematics Specialist Units 1,2
Test 2 2018

Section 2 Calculator Assumed

Vectors

STUDENT'S NAME _____

DATE: Thursday 29 March **TIME:** 30 minutes **MARKS**: 32

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser

Special Items: Three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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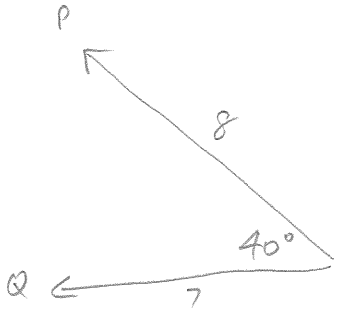
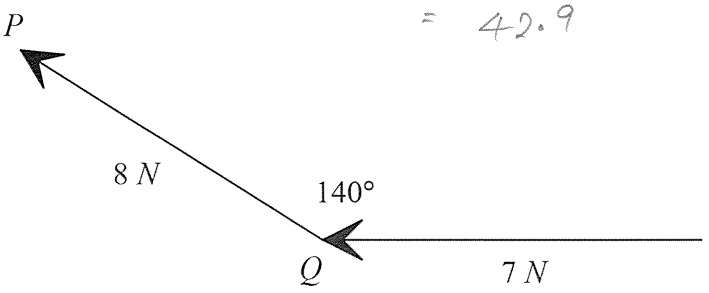
4. (7 marks)

(a) Determine $\underset{\sim}{p} \bullet \underset{\sim}{q}$ for the diagram shown. [2]

$p \cdot q = 7 \times 8 \times \cos 40^\circ$

$= 42.9$





(b) For the vectors $\underset{\sim}{c} = 8i - 15j$ and $\underset{\sim}{d} = 3i + 6j$ determine

(i) The scalar projection of $\underset{\sim}{d}$ on $\underset{\sim}{c}$ [2]

$|d| = \sqrt{45}$

ANGLE 125.4°

SCALAR PROJ $= \sqrt{45} \cos 125.4^\circ$

$= -3.88$

(ii) The vector projection of $\underset{\sim}{c}$ on $\underset{\sim}{d}$ [3]

$|c| = 17$

ANGLE 125.4°

VECT PROJ $= 17 \cos 125.4^\circ \left(\frac{3i+6j}{\sqrt{45}} \right)$

$= \begin{pmatrix} -4.4 \\ -8.8 \end{pmatrix}$

5. (8 marks)

A water bomber is to fly from Perth (P) to a bushfire near Northam (N). A wind is blowing with a velocity of $12i + 15j$ km/hr and $\overrightarrow{PN} = -15i + 65j$ km.

If the water bomber can maintain a still air speed of 270 km/hr, determine:

(a) the vector set on the plane to fly directly to the fire [5]

$\begin{pmatrix} a \\ b \end{pmatrix} + \begin{pmatrix} 12 \\ 15 \end{pmatrix} = \lambda \begin{pmatrix} -15 \\ 65 \end{pmatrix}$

$a = -12 - 15\lambda$

$b = -15 + 65\lambda$

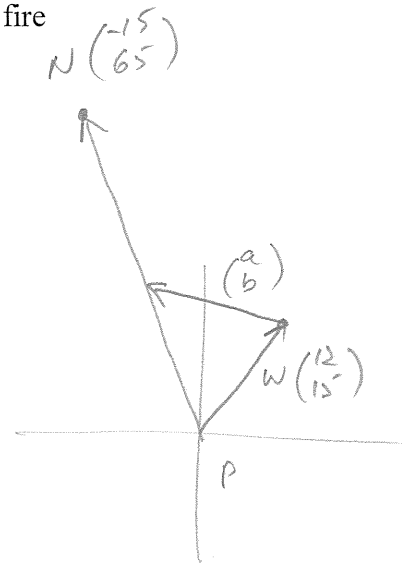
$a^2 + b^2 = 270^2$

$(-12 - 15\lambda)^2 + (-15 + 65\lambda)^2 = 270^2$

$\lambda = 4.22$

$a = -75.3$

$b = 259.3$



(b) the actual speed of the plane [2]

$\begin{pmatrix} -75.3 \\ 259.3 \end{pmatrix} + \begin{pmatrix} 12 \\ 15 \end{pmatrix} = \begin{pmatrix} -63.3 \\ 274.3 \end{pmatrix}$

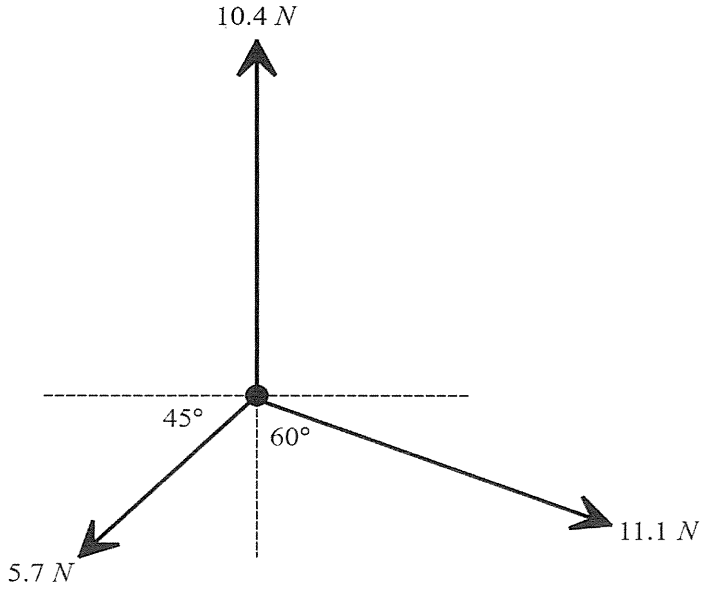
$\left| \begin{pmatrix} -63.3 \\ 274.3 \end{pmatrix} \right| = 281.5$ km/hr

(c) the time the plane takes to reach the bush fire [1]

$\frac{1}{\lambda} = 0.237$ HRS

6. (9 marks)

Three forces act on a body as shown in the diagram below.



(a) Determine the resultant vector in the form $ai + bj$. [4]

$[10.4 \angle 90^\circ] + [11.1 \angle (-30^\circ)] + [5.7 \angle (-135^\circ)]$

$= [5.58 \quad 0.82]$

$= 5.58\,i + 0.82\,j$

(b) Give the magnitude and true bearing of the resultant in (a). [3]

$\begin{pmatrix} 5.58 \\ 0.82 \end{pmatrix} = 5.64 \angle 8.35^\circ$

MAG 5.64 N

BEARING 082°

(c) If another force Q N acted on this system allowing it to be in equilibrium, what would be its magnitude and true bearing? [2]

MAG 5.64 N

BEARING 262°

7. (8 marks)

(a) Given vectors $A\begin{pmatrix}-3\\1\end{pmatrix}$, $B\begin{pmatrix}2\\-6\end{pmatrix}$, $C\begin{pmatrix}9\\-1\end{pmatrix}$ and $D\begin{pmatrix}4\\6\end{pmatrix}$, determine

(i) vector $\overrightarrow{AC}$ and vector $\overrightarrow{BD}$ [2]

$\overrightarrow{AC} = \underset{\sim}{c} - \underset{\sim}{a}$

$= \begin{pmatrix}9\\-1\end{pmatrix} - \begin{pmatrix}-3\\1\end{pmatrix}$

$= \begin{pmatrix}12\\-2\end{pmatrix}$

$\overrightarrow{BD} = \underset{\sim}{d} - \underset{\sim}{b}$

$= \begin{pmatrix}4\\6\end{pmatrix} - \begin{pmatrix}2\\-6\end{pmatrix}$

$= \begin{pmatrix}2\\12\end{pmatrix}$

(ii) $\overrightarrow{AC} \bullet \overrightarrow{BD}$ [2]

$\begin{pmatrix}12\\-2\end{pmatrix} \cdot \begin{pmatrix}2\\12\end{pmatrix} = 0$

(b) Give a geometric interpretation of (a). [2]

VECTORS PERPENDICULAR

(c) For any 3 vectors, is it possible to determine $\underset{\sim}{a} \bullet \underset{\sim}{b} \bullet \underset{\sim}{c}$? Explain your answer. [2]

NO

$a \cdot b$ = SCALAR

CANNOT DO SCALAR $\cdot \underset{\sim}{c}$